QUANTUM UNIVERSE: GEOMETRY & TOPOLOGY.
FINAL EXAM 2015/16

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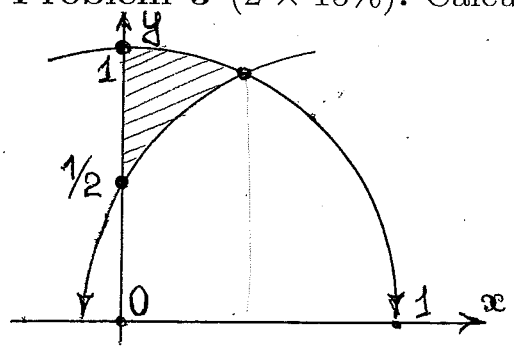
Problem 1 (10%). Using the polar coordinates $\bigl(r \geqslant 0,\ \varphi \in [0, 2\pi)\bigr)$, calculate the scalar curvature R of the Euclidean plane $\mathbb{E}^2$ with the line element $\mathrm{d}\ell^2 = \mathrm{d}r^2 + r^2\,\mathrm{d}\varphi^2$.

Problem 2 (20%). Prove the identity $\Gamma^i_{ki} = \dfrac{1}{2g}\dfrac{\partial g}{\partial x^k}$, where $g = \det(g_{\mu\nu})$ and the symmetric affine connection $\{\Gamma^a_{bc}\}$ is associated with the Riemannian metric $g_{\mu\nu}$ by the formula

$$\Gamma^k_{ij} = \tfrac{1}{2} g^{k\ell}\left(\frac{\partial g_{\ell j}}{\partial x^i} + \frac{\partial g_{i\ell}}{\partial x^j} - \frac{g_{ij}}{\partial x^\ell}\right).$$

Problem 3 (2 × 15%). Calculate the angles of the triangle in the hyperbolic plane $\mathbb{H}^2$ with the metric $\mathrm{d}s^2 = (\mathrm{d}x^2 + \mathrm{d}y^2)/y^2$, $y > 0$ (see figure); the sides of the triangle are segments of the geodesics in $\mathbb{H}^2$.



- Prove [e.g., by an explicit calculation] that the sum of these three angles is strictly less than π.

Problem 4 (20%). Find the law of material point's inertial motion along the side surface of a circular cylinder of radius $r > 0$ in $\mathbb{E}^3$.

Problem 5 (20%). Prove that the scalar curvature R of a two-dimensional real Riemannian manifold M^2 with a symmetric Riemannian connection associated with the metric $g_{\mu\nu}$ is related to just one component of the Riemann tensor $R_{ij,k\ell}$ on M^2 by the formula $R = 2R_{12,12}/\det(g_{\mu\nu})$.

Date: March 31, 2016.
Do not postpone your success until 23 June. GOOD LUCK!